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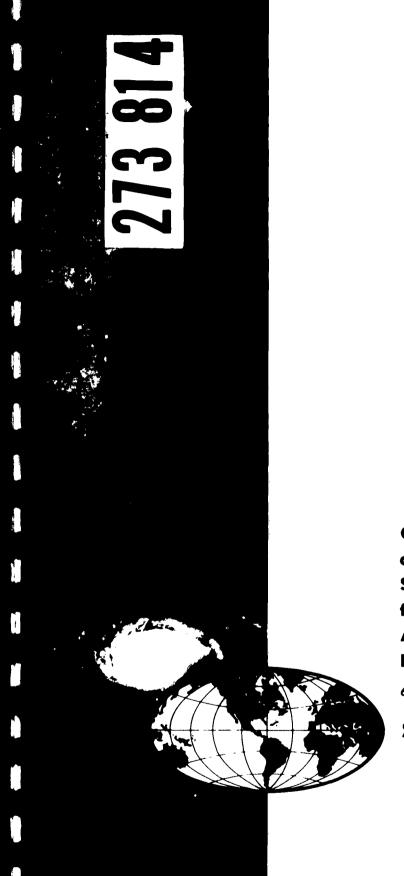
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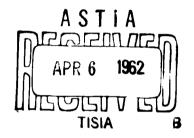
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COMPARISON OF IR VIDICON and
SENSITIVE VISIBLE LIGHT CAMERA TUBE for application in
AIRBORNE-TV GUIDANCE
RECONNAISSANCE SYSTEMS

61 SPC-7

SPACE SYSTEMS



COMPARISON OF IR VIDICON and SENSITIVE VISIBLE LIGHT CAMERA TUBE for application in AIRBORNE-TV GUIDANCE/RECONNAISSANCE SYSTEMS

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Santa Barbara, California

SUMMARY

The basic principles of radiation from a heated surface are examined. Equations are developed that provide a means of determining the performance of an infra red or visible light TV system in terms of the camera tube, quantum efficiency, the noise current and the lens diameter and F number.

The over-all system performance is given in terms of RMS current signal-to-noise ratio, and frame repetition rate at a signal-to-noise ratio of 2. In the case of the visible light sensitive tube, the relationship between object illumination and signal-to-noise ratio, and resolution vs. lens diameter are tabulated and plotted.

This study indicates that the IR Vidicon would have greater application for use in the TV Guidance/Reconnaissance Systems than the visible light sensitive camera. The advantages of the IR Vidicon over the visible light tube are as follows:

- 1. Can operate independent of day or night illumination.
- 2. The lens aperature size requires less continuous adjustment since it is independent of natural illumination.
- 3. Although the IR Vidicon does not pessess the resolution capability of the Z5294 Image Orthicon (500 vs. 700 lines), it has sufficient resolution for the application. Further, for the required range of 100,000 feet the IR Vidicon can achieve sufficient resolution with an optical lens of reasonable size.

Lambert's Cosine Law of Emmission

Surface brightness of an illuminated (or heated) surface is defined as the ratio of the total flux (or power) emitted in the particular direction from which the surface is viewed to the "projected" area of the surface as viewed from the same direction.

The total amount of power emitted from the surface is the integral of the brightness taken over the solid angle of a hemisphere. This integration can only be performed if the law by which brightness varies with the direction from which the surface is viewed is known. A heated "black body" surface appears equally bright when viewed normally and when viewed at an angle with suitable instruments. Such a surface therefore must emit equal power densities in all directions.

In figure 1, let P_n be the power radiated in the direction normal to the surface and P_n the power radiated in the direction to the normal.

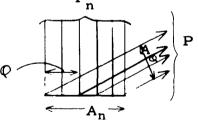


Figure 1

⁽¹⁾ E. Hausmann and E. P. Slack, "Physics", D. Van Nostrand Co., 1944, pp 453-459.

Let A be the area of the radiating surface. Its "projected" area as viewed from the direction a is

$$A_{\mathbb{Q}} = A_{\mathbb{n}} \cos \mathbb{Q} \tag{1}$$

and since the brightness is the same in both directions

$$\frac{P_n}{A_n} = \frac{P_0}{A_0} = \frac{P_0}{A} / \cos Q$$
From which $P_0 = P_n \cos Q$ (2)

This is Lambert's Cosine Law of Emission and applies rather generally to radiation from heated black body surfaces.

Surfaces which obey this law are known as "Lambert Surfaces."

Intensity of Radiation From a Surface 2

Let ds be a small Lambert Surface element of a radiating body. Describe about ds a hemisphere of radius r and let dB located at P be a small element of surface of the hemisphere.

Let the radius OP to this element of surface make an angle on to the normal ON of ds. (see figure 2)

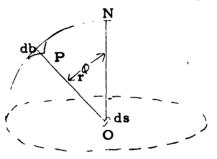


Figure 2

Let the intensity of radiation in the direction ON normal to the surface

$$ds = I_0 \text{ watts/steradian cm}^2$$
 (3)

then from Lambert's cosine Law, the intensity of radiation in the direction $\boldsymbol{\varrho}$

$$I_{\mathcal{O}} = I_{0} \cos \mathcal{O} \quad \text{watts/steradian cm}^{2}$$
 (4)

(2) based on: F. K. Richtmyer and E. H. Kennard, "INTRODUCTION TO MODERN PHYSICS", McGraw-Hill Book Co., 1947, pp 141-142

Since $\frac{dB}{r^2}$ is the element of solid angle which the element of area subtends at O, the power incident on dB is

$$W_{dB} = I_6 \cos Q \frac{dB}{r^2} \text{ watts/cm}^2$$
 (5)

In order to determine the total power radiated by ds, we must integrate the power incident on each element of area on the hemisphere surface.

Referring to figure 3, let us choose as an element of surface area of the hemisphere, a ring of width r do and of length $2\pi r \sin Q$.

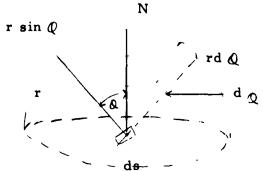


Figure 3

the surface area of the ring therefore is $2\pi r^2 \sin Q d Q$.

The power incident upon this ring is from equation (5)

$$W_{r} = I_{o} \cos \theta \frac{2\pi r^{2}}{r^{2}} \sin \theta dQ = 2\pi I_{o} \cos Q \sin Q dQ \qquad (6)$$

Integrating from Q = 0 to $Q = \frac{\pi}{2}$ we get the total power radiated

W, from the element of radiating surface ds

$$W_{t} = 2\pi I_{0} \int_{0}^{\frac{\pi}{2}} \cos \theta \sin \theta \, d\theta$$

$$= \pi I_{0} \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta$$

$$= \frac{-\pi I_0}{2} \qquad \left[\cos 2\mathbb{Q}\right]_0^{\frac{\pi}{2}}$$

$$= \pi I_0 \text{ watts/cm}^2$$
 (7)

so that

I

$$I_{o} = \frac{W_{t}}{\pi} \text{ watts/steradian cm}^{2}$$
 (8)

The power incident on a surface area dB at an angle Q to the normal of the surface is therefore

$$WQ = \frac{W_t \cos Q}{\pi} \frac{dB}{r^2}$$
 watts/cm² (9)

Equation (5) shows that the maximum power is incident on the surface of a detector when the detector is normal to the radiating

surface, but brightness, which is radiated power in a given direction divided by projected area of the radiating surface, is constant with angle.

Black Body Radiation 3

The total amount of power radiated per unit area from a black body surface is given by

$$W_t = \int_0^\infty W d \lambda = \sigma T^4 \text{ watts/sq. cm}$$
 (10)

where W_t is the power radiated in watts/sq. cm

O is Stefan-Boltzman's constant = 5.679 X 10⁻¹² watts/cm² T⁴

T is the absolute temperature in degrees Kelvin

For example if the surface temperature of a black body is 300° K.

$$W_t = 5.679 \times 10^{-12} \times 81 \times 10^8$$

= 4.6 \times 10^2 watts/sq. cm

If the surface is other than a black body one has to include the emissivity factor f which is defined as

emissivity = energy radiated from surface in question energy radiated from a perfect black body,

when both surfaces are of equal area and at equal temperature.

(3) M. M. Reynolds, R. J. Corruccina, M. M. Faulk, "AMERICAN INSTITUTE OF PHYSICS HANDBOOK;" McGraw-Hill Book Co., 1951 pp 6-64-6-67

So Equation (10) becomes

$$W_t = \mathcal{C} \mathcal{T}^4 \text{ watts/sq. cm}$$
 (11)

Power Radiated Per Unit Wavelength Interval 3

The power radiated per unit wavelength interval λ by a unit area of a black body at a temperature of T $^{\bullet}$ K is given by the Planck formula

$$W(\lambda, T) = \frac{C_1}{\lambda^5 \left(-C_2/\lambda T_{-1}\right)} \text{ watts cm}^{-2} \mu^{-1}$$
 (12)

The maximum value of $W(\lambda, T)$ is given by

$$W_{max}(T) = 1.290 \times 10^{-15} T^5 \text{ watts/cm}^2 \mu^1$$
 (13)
where μ is the wavelength in microns = 10^{-4} cm

Solutions of equations (10), (12), and (13) are tabulated in the referenced (3) article for W_t and $W_{max}(T)$ covering the temperature range from 1°K to 1940°K and for $W(\lambda, T)$ ranging from $\lambda T = 0.050$ to 2.0.

These tables provide a ready means for determining the total power radiated and the spectral distribution of the radiated energy in any wavelength band of interest. Plots of $W_{max}(T)$ and $\frac{W(\lambda T)}{W_{max}(T)}$ are given in figures 4 and 5 respectively.

As an example of the use of these tables, let us assume

that we have a target of area 1 sq. cm at a temperature of 300°K and we wish to determine the amount of energy radiated in the 3.5 - 4.1μ band.

For this particular example we are not concerned with the total amount of power radiated; we are only concerned with $W_{max}(T)$ and $W(\lambda, T)$.

From the table we see that W max for T = 300 °K = 3.134 X 10^{-3} watts per sq. cm per μ . This is the amount of power in a 1 μ band centered around the wavelength of maximum radiation. The next step is to calculate the product λ T, if we take λ as the average of 3.5 to 4.1 μ i.e., 3.8 μ , we get

$$\lambda$$
 T = 3.8 X 10⁻⁴ (300) (since 1μ = 10^{-4} cm)
= 11.4 X 10^{-2} = 0.114 cm-degrees

The nearest value to this in the table is 0.115, the corresponding value of $\frac{W(\lambda, T)}{W_{max}(T)} = 5.350 \times 10^{-2}$

so that

$$W(\lambda, T) = 5.35 \times 10^{-2} \times 3.134 \times 10^{-3}$$

= 1.68 × 10⁻⁴ watts cm⁻² μ ⁻¹

This is the amount of power in a l μ band centered around a wavelength of 3.8 μ .

Since the spectral distribution curve of energy vs. wavelength is reasonably flat for a temperature of 300°K, we are

justified in saying that the amount of power in a band 0.6μ wide

 $= 0.6 \times 1.68 \times 10^{-4}$

ز

= $10.08 \times 10^{-5} \text{ watts cm}^{-2}$.

Relation Between Lens Diameter, Detector Surface and Object Area

Let us consider a lens of diameter D and a detector target image I arranged as in figure 6 with respect to the object O, that we wish to detect

LENS, DIAMETER: D

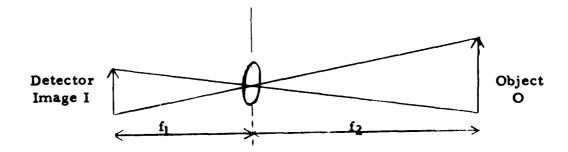


Figure 6

Let f_1 be the distance between the image and the lens; also let this distance be the focal length of the lens, let f_2 be the distance between the object and the lens.

The linear magnification 4

$$M_{L} = \left(\frac{f_1}{f_2}\right) = \frac{\text{image size}}{\text{object size}}$$
 (14)

The area magnification of the lens

$$MA = \left(\frac{f_1}{f_2}\right)^2 = \frac{\text{area image}}{\text{area object}}$$
 (15)

The lens "F" number is approximately given by

$$\mathbf{F} = \frac{\mathbf{f_1}}{\mathbf{D}} \tag{16}$$

If the object is radiating a power of Io watts/per steradian - cm² normal to its surface, our transmission medium is a vacuum and we view the object from a point that is normal to the object surface, we can say that the power intercepted by the lens

$$W_{L} = \frac{\text{area lens } I_{D}}{f_{2}^{2}} \quad \text{watts}$$

$$= \frac{\pi D^{2}}{4} \qquad \frac{I_{O}}{f_{2}^{2}} \quad \text{watts}$$
(17)

If we further assume that there are no losses in the optical system, ⁽⁴⁾Physics, Hausman and Slack, D. Van Nostrand Co., February, 1944, page 508.

then the power in the image is equal to that of the lens

where
$$\frac{Q}{Q}$$
 the image power density,
$$W_{DI} = \frac{\pi D^2}{4} \frac{I_{O}}{(f_2)^2} \frac{(f_2)^2}{(f_1)^2} \text{ watts/sq. cm}$$

$$= \frac{\pi D^2}{4} \frac{I_{O}}{(f_1)^2} \text{ watts/sq. cm}$$
(18)

but

$$\left(\begin{array}{c} \mathbf{D} \\ \hline \mathbf{f_1} \end{array}\right)^2 = \frac{1}{\mathbf{f}^2}$$

80

$$W_{DI} = \frac{\pi I_{O}}{4 F^{2}} \tag{19}$$

and from equation (8)

$$I_o = \frac{W_t}{\pi}$$

80

$$W_{DI} = \frac{W_t}{4 F^2} \text{ watts/sq. cm}$$
 (20)

WDI is the image power density in watts per sq. cm of image area.

Since the detectors we are concerned with here are Infrared camera tubes we desire to know the relationship between the camera tube parameters, the lens F number and the power radiated by the object.

The IR vidicon tube at present under development by the General Electric Company has the following predicted characteristics:

- 1. Spectral response: 1-4 μ
- 2. Quantum efficiency 5 X 10⁻⁴
- 3. RMS noise current 2 X 10⁻¹⁰ amps
- 4. Resolution 500 lines
- 5. Image target surface area: 25 mm X 20 mm
- 6. Frame rate 1/30 seconds

The quantum efficiency is defined as

$$q = \frac{\text{electrons emitted/cm}^2}{\text{photons incident/cm}^2}$$
 (21)

so for

$$T_{ig} = 5 \times 10^{-4}$$

electrons emitted/cm² = 5 X 10⁻⁴ Photons incident/cm²

We will recall that the energy in

1 photon =
$$6.62 \times 10^{-27} f ergs$$
 (22)

where f is frequency in cps, and that an electron's charge

$$eg = 1.602 \times 10^{-19}$$
 coulombs
= 1.602×10^{-19} amp-secs (23)

and since

and

$$f = \frac{C}{\lambda} = \frac{3 \times 10^{10}}{\lambda^{cm}} cps$$

the number of photons per sec. per cm² emitted by a surface

$$P_{h} = \frac{W \lambda \times 10^{7} \quad \lambda}{6.62 \times 10^{-27} \quad 3 \times 10^{10}} \quad Photons/sec. cm^{2}$$

$$= 5.05 \times 10^{22} \quad W_{\lambda} \quad \lambda$$
(24)

where $W\lambda$ is the total power radiated in watts per sq. cm of object surface area in the band of interest centered at wavelength λ .

The photon density per sec. on the image surface is from equations (20) and (24).

$$PhDI = \frac{5.05 \times 10^{22} \text{ W}}{4 \text{ F}^2}$$
 photons/sec. cm² (25)

and the number of electrons emitted

$$= \frac{5.05 \times 10^{22} \text{ W}}{4 \text{ F}^2}$$

$$= \frac{25.25 \times 10^{18} \text{ W} \lambda^{\frac{1}{2}}}{4 \text{ F}^2} \text{ sec. cm}^2$$
 (26)

So the current

$$I_{S} = \frac{25.25 \times 10^{18} \times 1.602 \times 10^{-19} \text{ W} \lambda^{\frac{1}{2}} / \text{cm}^{2}}{4 \text{ F}^{2}}$$

$$= \frac{4.05 \text{ W} \lambda^{\frac{1}{2}} \lambda^{\frac{1}{2}} \text{ amp/cm}^{2}}{4 \text{ F}^{2}}$$
(27)

Equation (27) gives the RMS current output from the detector image for each square cm of its surface area. This current is given in terms of the lens F number, the wavelength of radiation, and the radiated power for each square cm of object area. We are interested primarily in obtaining sufficient contrast between an element of resolution within the object area of view and the background which comprises the remaining portion of the area of view. Further we wish to attain this contrast in the presence of the noise current, In generated within the camera tube. Since the amount of energy radiated by an element of resolution and by the background are functions of their respective temperatures (neglecting their respective emissivities), the amount of contrast obtainable will depend on the temperature difference between them.

If we set our objective for a contrast ratio of 5%, which

we understand is a reasonable figure, then this would correspond to a temperature difference of approximately 10°K in 300°K between an element of resolution and an element of background. (This value was derived from equations (12), (13) and (27) for temperatures of 310° and 300° Kelvin).

It is of interest to note that the element of resolution may be at a higher, or lower temperature, than an element of background. If it is of a higher temperature, it will show up light against a darker background, and if of a lower temperature, it will show up dark against a lighter background.

The noise current I_n in the IR tube will be superimposed on both the element of resolution and the background element. Depending on the amplitude and phase relationship of the noise current and respective element currents, the noise will tend to reduce or increase the brightness of the resolution elements.

It is desirous therefore that the signal-to-noise ratio be made as large as possible.

A signal-to-noise ratio of 4 on a power basis, between an element of resolution and the IR tube noise is considered to be a reasonable design objective in an IR detection system. This corresponds to an RMS signal-to-noise current ratio of 2.

Since the desired signal that we are concerned with is due to an element of resolution that is generated in the presence of other signals that are generated by the background elements, we can consider the desired signal to be of peak-to-peak amplitude. If we make the assumption that this peak-to-peak signal approximates a sine wave we can say that

$$\mathbf{w} \lambda = \frac{+ \mathbf{w} \lambda \mathbf{R} + \mathbf{w} \lambda \mathbf{B}}{2 \sqrt{2}}$$
 (28)

If we denote the desired signal current due to the difference from the image area temperatures of the object elements of resolution as $I_{\mathbf{R}^{\prime}}$. We can write

$$I_{R} = \frac{4.05}{8\sqrt{2}} \frac{(\frac{1}{2} \text{ W } \lambda \text{ R} + \text{ W } \lambda \text{ B}) \lambda}{F^{2}}$$

$$= 0.356 \frac{(\frac{1}{2} \text{ W} \lambda \text{ R}) + \text{ W } \lambda \text{ B}}{F^{2}} \lambda \text{ amps/cm}^{2}$$
(29)

We know from the IR Vidicon characteristics that the RMS noise current is based on a resolution of 500 lines and a frame repetition rate $f_{\mathbf{r}}$ of 30 cps. Since this noise current is largely due to Johnson noise we can say that

$$I_n = 2\sqrt{\frac{KTB_v}{R}} \quad amps \tag{30}$$

where

K is Boltzman's constant
= 1.38 X 10⁻²³ Joules/degrees Kelvin

T is absolute temperature in degrees Kelvin R is the resistive component of IR tube in which the noise is generated B_V is the video bandwidth in cps.

Now

$$B_{v} = \left(\frac{R_{h}}{2}\right)^{2} \quad fr \quad b \quad cps \tag{31}$$

where

fr is the frame repetition rate in cps \mathbf{R}_h is the horizontal resolution in lines \mathbf{b} is the image area aspect ratio

Since all the terms in equations (30) and (31) can be considered as constants which are inherent in the tube design, except for fr and B_V (and B_V is a function of fr) we can say that $I_n = k\sqrt{fr}$. This enables us to express the noise current I_n as a function of frame repetition rate; so we get

$$I_{n} = \frac{2 \times 10^{-10} \sqrt{fr}}{\sqrt{30}}$$

$$= 3.66 \times 10^{-11} \sqrt{fr} \text{ amp}$$
(32)

So on combining equations (29) and (32), we get the RMS signalto-noise ratio on a current basis.

$$S/N = \frac{0.356 (-W) R + W B R}{3.66 \times 10^{-11} \sqrt{fr}} = \frac{9.75 \times 10^{9} (-W) R + W B R}{\sqrt{fr}}$$
(33)

and for S/N = 2

$$\frac{(-W) \chi_R + W}{\sqrt{fr} + W} \chi_B \chi_A = 2.05 \times 10^{-10}$$
(34)

Up to this point we have assumed that the transmission of energy from the heated body object surface to the lens took place in a vacuum. We further assumed that the efficiency of the optical system was 100%.

Let us assign an attenuation factor to represent transmission losses in the atmosphere. The numerical value of will be a function of transmission path length which is intrinsic in the F² term in equation (33) and the "weather conditions" in the atmosphere through which the transmission takes place. It should be pointed out that the subject of infra-red attenuation in the atmosphere is a highly complex one, and only very limited data exists at present particularly for high altitudes and over long path lengths. will be assigned a value of unity for the case of transmission through a vacuum. We shall account for the losses through our optical system by assigning an efficiency factor of say 0.9. Therefore, we shall rewrite equation (34) as follows:

⁽⁵⁾ J. N. Howard, "The Transmission of the Atmosphere in the Infrared", Proc. I. R. E. Vol. 47, No. 9, Sept. 1959, pp 1451-1457.

$$\frac{(\stackrel{+}{-}W) R \stackrel{+}{+} W R \stackrel$$

and

$$\mathbf{F} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

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$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{g} = \begin{bmatrix} (\frac{1}{2} \mathbf{W} \mathbf{\chi} \mathbf{R} + \mathbf{W} \mathbf{\chi} \mathbf{B}) \mathbf{\chi} & \gamma & \mathbf{p} \end{bmatrix}^{\frac{1}{2}}$$

It will be recalled from equations (16) and (15) that

$$\mathbf{F} = \frac{\mathbf{f_1}}{\mathbf{D}}$$

and

$$(f_1)^2 = \frac{\text{area image}}{\text{area object}} (f_2)^2$$

it follows also that

$$(f_1)^2 = \frac{\text{area of image element of resolution}}{\text{area of object element of resolution}} \begin{pmatrix} f_2 \end{pmatrix}^2$$

$$= \frac{A_1}{R^2 h \ b \ Aor} = \begin{pmatrix} f_2 \end{pmatrix}^2 \qquad (37)$$

where $A_{\rm I}$ is the area of the IR tube image and $A_{\rm Or}$ is the area of the object element of resolution. Values of $f_{\rm I}$ for $f_{\rm 2}=10,000$ feet and 100,000 feet as a function of $\sqrt{A_{\rm Or}}$ are tabulated in table I. The results of table I are plotted in figure (7).

TABLE I

Lens focal length vs. object resolution: IR Vidicon

√Aor feet	f ₂ = 10K ft. f ₁ = cm	f ₂ = 100K ft. f ₁ = cm
10	4	40
20	2	20
25	1.6	16
30	1. 33	13. 3
40	1.0	10
50	0.8	8.0
60	0.67	6. 7
70	0.57	5.7
100	0.4	4
		ļ

Values of F, and lens Diameter D, derived from (36) and Table I, for $f_2 = 100,000$ ft., $\sqrt{A_{0r}} = 50$ feet, $\ll = 1$ (vacuum transmission) $\gamma = 1$, background temperature = 300°K, resolution element temperature = 310°K, $\lambda = 3.8 \, \text{M}$, spectrum = 3.5-4.1 and S/N = 2 are tabulated in table II. The results are plotted in figure (8)

TABLE II*

Lens F no. and diameter vs. frame rate,

Frame Rate cps	Lens F number	Lens Diameter cm
5	7. 1	1.1
10	6. 0	1.3
15	5. 4	1.5
20	5. 06	1.6
25	4. 78	1.7
30	5. 55	1.76

^{*} range (f₂): 100,000 ft.

object resolution element: 50 ft.

S/N = 2, (current basis)

Horizontal resolution: 500 lines

= 1 (vacuum transmission)

 $\eta_0 = 1$, $\lambda = 3.8 \mu$, Spectrum: 3.5 - 4.1 μ

Re solution element temperature: 310°K

Background temperature: 300°K

Detector: IR Vidicon

Table III shows the variation in signal to noise ratio as a function of lens F number and diameter for a constant frame frequency of 30 cps.

TABLE III*

S/N vs. Lens F no. and Diameter for fr constant at 30 cps.

Lens F Number	Lens Diameter cms	S/N R MS Current
1	8	42
1.33	6	23. 3
2	4	10.5
3	2. 67	4. 67
4	2	2. 63
4. 55	1.76	2

^{*} range $f_2 = 100,000 \text{ ft.}$

Object resolution element = 50 ft.

Ho rizontal resolution = 500 lines

$$\eta_o = 1$$
, $\lambda = 3.8 \,\mu$, spectrum 3.5 - 4.1 μ ,

Resolution element temperature = 310°K

Background temperature = 300°K

Detector: IR Vidicon

The results of Table III are plotted in figure 9.

Figure 10 shows the relationship between lens diameter and frame repetition rate for a RMS current signal to noise ratio of 2 under clear weather conditions. An optical efficiency of 90% has been assumed.

Visible Light Image Orthicon Detector

As a basis of comparison between infrared and visible light detectors we shall now consider the General Electric type Z5294 Image Orthicon.

Based on information recently received 6 the characteristics of the Z5294 tube are as follows:

Quantum Efficiency 5-10%, Accelorator Gain: 12 RMS noise current for a video bandwidth of 4 mc: approximately 2-2. 5 X 10⁻¹⁰ amps

Spectral Response: (see fig. 11)

Relative energy %	Wavelength Angstroms
2	3000
80	3500
95	4000
90	4500
100	5500
80	6000
10	7000

Frame repetition rate: 30 cps

Image Size: across diagonal; 1.25 - 1.6 inches

Aspect ratio: 4:3

Note:

The noise current is a function of beam current and also the effective bandwidth of amplification following the tube.

Characteristics of Z5294 provided by R. Reddington, Research Laboratory, Schenectady, N. Y.

At low light levels the beam current would have to be increased by a factor of 10 to 30 which would result in a very high value of noise current.

On the basis of the noise current being of the order of $2-2.5 \times 10^{-10}$ amps. for a video bandwidth of 4 mc. and with a frame rate of 30 cps we can say the horizontal resolution

$$R_{h} = \sqrt{\frac{2 B_{v}}{fr}}$$
 (38)

so let us say 500 lines and assume the upper value of noise current i. e. 500 lines.

Since the diagonal distance across the corners of the image area is between 1.25 to 1.6 inches let us choose a nominal value of 1.5 inches (see fig. 12 below)

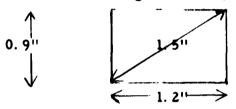


fig. 12

Z5294 Image Area Geometry

It follows that since the aspect ratio is 4:3 the width of the image is 1.2 inches and the height is 0.9 inches. The surface area of the image therefore is 1.08 square inches.

From the value of noise current assumed of 2.5 \times 10⁻¹⁰ amps, we can write by utilizing equation (32)

$$I_{n} = \frac{2.5 \times 10^{-10} \sqrt{fr}}{\sqrt{30}}$$

= 4.5
$$\times 10^{-11} \sqrt{\text{fr}} \text{ amps}$$
 (39)

Let us now consider the lens requirement utilizing the Z5294 tube for viewing an object of resolution of 50 ft at a range of 100,000 ft. This will give us a basis of direct comparison between the IR Vidicon and the Z5294 Image Orthicon. Let the

object area of view be illuminated by a visible light flux of E lumens per cm². Now 1 lumen = 1.5 \times 10⁻³ watts and we will recall that 1 photon = 6.62 \times 10⁻²⁷ f ergs, where f equals $\frac{c}{1}$.

$$E_{p} = \frac{E \times 10^{7} \times 1.5 \times 10^{-3} \lambda}{6.62 \times 10^{-27} \times 3 \times 10^{10}}$$
 (40)

where 1 is the wavelength in cms.

whence
$$E_p = 7.55 \times 10^{19} E_{\lambda}$$
 (41)

It will be noted that E and hence E_p is the total illumination per unit area of the object. If we consider the object to reradiate light energy as a diffused source then Lambert's Cosine Law of Emission will apply. Further the amount of light energy reradiated will depend on R, the reflection coefficient of the object surface.

If we further use the same reasoning as for the heated surface in the infrared radiation case (see pages 9 and 10) we can say by analogy with equation (28) that the flux intensity that generates the desired signal,

$$E_{pr} = \frac{E(\pm R_r + R_B) \times 7.55 \times 10^{19} \lambda}{2\sqrt{2}}$$
 (42)

where $\mathbf{E_{pr}}$ is the flux intensity in photons per sec - cm² $\mathbf{R_r}$ is the reflection coefficient of the object element of resolution $\mathbf{R_B}$ is the reflection coefficient of the element of background. When the surface of the detector lens is located at a point that is normal to the illuminated surface, the total flux intercepted by the lens in photons/sec.

$$PL = \frac{AL \ E \ (\pm R_r \ \bar{\pm} R_R) \ X \ 7.55 \ X \ 10^{19} \ \lambda}{\pi^2 \ (f_2)^2 \ 2 \ \sqrt{2}}$$
 (43)

where A_L is the area of the lens in cms².

f'z is the range in cms.

The illumination of the detector image area

$$E_{PRI} = \frac{\left(\frac{f_2}{f_1}\right)^2}{\frac{4\pi (f_2)^2}{2\sqrt{2}}} = \frac{(\pm R_r \mp R_B) \times 7.55 \times 10^{19} \lambda}{2\sqrt{2}}$$

$$= \frac{E(\pm R_{\perp} \mp R_{B})\lambda}{F^{2}} \times 6.65 \times 10^{18} \text{ photons/sec cm}^{2}$$
(44)

and the number of electrons emitted by the photo cathode = $\mathbf{E}_{\mathbf{PRI}} \Omega_{\mathbf{q}}$ The number of electrons impinging on the target = $\mathbf{E}_{\mathbf{PRI}} \Omega_{\mathbf{q}}$ G where G is the accelerator gain

$$\therefore I_{R} = 1.07 \quad \underline{E} \left(\pm R_{p} \mp R_{R} \right) \lambda \qquad \mathcal{D} \text{ qG amps}$$

$$F^{2}$$
(45)

and on combining equations (39) and (45), the RMS signal to current ratio S/N = $\frac{2.34 \times 10^{10} E(\pm R_r \mp R_B) \lambda \eta_q G}{\sqrt{fr}}$ (46)

and taking the transmission loss factor \bigcirc , and the optical transmission efficiency into account, we can say

$$S/N = \frac{2.34 \times 10^{10} \text{ E}(\pm R_r \mp R_R) \lambda \Omega \gamma_0 \gamma_q G}{\sqrt{\text{fr}} \text{ F}^2}$$
(47)

and for a S/N = 2,

$$\frac{E(\pm R_{\perp} \mp R_{\rm R}) \lambda (\gamma \circ \gamma \circ q G = 8.55 \times 10^{-11})}{\sqrt{\text{fr}} \qquad F^2}$$
(48)

and
$$F = \left[\frac{E(\pm R_r \mp R_B) G\lambda \Omega \eta_o \eta_q}{8.55 \times 10^{-11} \sqrt{fr}}\right]^{1/2}$$
 (49)

and from equation (37)

$$f_1^2 = A_T$$
 $R_h^2 B$ Aor

and for the Z5294 tube

$$A_I = 1.08 \text{ sq. inches}$$

$$b = 4/3$$

$$R_{H} = 500 \text{ lines}$$

so we get
$$f_1 = 4.57 \times 10^{-3} \frac{f_2}{\sqrt{A_{or}}}$$
 cms

Values of f_1 for $f_r = 10,000$ ft. and 100,000 ft. as a function of $\sqrt{A_{or}}$ are tabulated in Table IV. The results of table IV are plotted in figure 13,

TABLE IV

Lens focal length vs. object resolution, Z5294 tube

Object Resolution	$f_2 = 10 \text{ K ft.}$	$f_2 = 100 \text{ K ft.}$
$\sqrt{A_{or}}$ feet	f ₁ cms	f _l cms
25	1. 83	18. 3
50	0. 915	9. 15
75	0. 61	6. 1
100	0. 4 6	4. 6

Values of F, and lens diameter D vs. frame rate derived from equation (49) and Table IV for f_2 = 100,000 ft., an object resolution of 50 ft., $\alpha = 1$, $\alpha = 0.1$, $\alpha = 1.2$, $\alpha = 1$, illumination of 2.8 $\times 10^{-5}$ lumens/ft². A background reflection coefficient R_B of 0.1, an object resolution coefficient R_r of 0.2 (rusty tank),

and $\lambda = 0.5 \,\mu$ (center of visible spectrum) and a S/N of 2 are tabulated in table V.

The results are plotted in fig. (14).

TABLE V

Lens F number of diameter D vs. frame rate, Z5'294 for S/N = 2

Frame rate	Lens F number	Lens diameter scms
срв		
5	0. 031	295
10	0. 026	351
15	0.024	386
20	0. 022	420
25	0. 021	444
30	0. 0197	464

Table VI shows the variation in S/N on a RMS current basis as a function of lens F number and diameter for a constant frame frequency of 30 cps.

TABLE VI*

S/N vs. Lens F number and diameter for $f_r = 30$ cps Z5294 Image Orthicon, Starlight Illimination

Lens F	Lens Diameter	
Number	cm	S/N
1 X 10 ⁻³	9150	776
2. 79 X 10 ⁻³	3280	100
1 X 10 ⁻²	915	7. 76
1.97 X 10 ⁻²	464	2
1 X 10 ⁻¹	91.5	7.76 X 10 ⁻²
1	9. 15	7. 76 X 10 ⁻⁴

* Illumination 2.85 X 10⁻⁵ lumens/ft²

 λ = 0.5 μ , Range: 100,000 ft.

Lens focal length: 9.15 cms

Resolution 500 lines

 $R_r = 0.2$, $R_B = 0.1$, $\gamma \circ = 1$, $\gamma = 1$

The results of table VI is plotted in figure 16. Table VII shows signal to noise ratio vs. illumination for a constant frame rate of 30 cps under clear weather transmission.

TABLE VII*
S/N vs. Object Illumination

S/N	Lumens - cm ² Lumens 1.44 X 10 ⁻⁹ 1.335 X 2.88 X 10 ⁻⁴ 2.77 X 1 5.76 X 10 ⁻⁴ 5.35 X 1 7.2 X 10 ⁻⁴ 6.68 X 1 1.152 X 10 ⁻³ 1.04	Illumination
RMS Current	Lumens - cm ²	Lumens - ft ²
2	1.44 × 10 ⁻⁹	1. 335 X 10 ⁻¹
4	2.88 X 10 ⁻⁴	2.77 X 10 ⁻¹
8	5.76 X 10 ⁻⁴	5. 35 X 10 ⁻¹
10	7.2×10^{-4}	6.68 X 10 ⁻¹
16	1.152 X 10 ⁻³	1.04
20	1.44 X 10 ⁻³	1. 335
50	3.6 X 10 ⁻³	3. 34

^{*} 7/0 = 0.9, C = 0.6, $f_r = 30$

 $R_r = 0.2, R_B = 0.1$

Lens F number = 1, resolution = 50 ft.

$$\lambda = 0.5 \mu$$

The results obtained from table VII are plotted in fig. 17.

Table VIII shows lens diameter required for variation in frame repetition rate for a S/N of 2 under moonlight illumination.

TABLE VIII*
Frame rate vs. Lens Diameter

rame Rate	F	Lens Diameter
cps	Number	cms
5	0. 58	15.7
10	0. 49	18.8
15	0.44	20.7
20	0.412	22 . 2
25	0. 39	23. 6
30	0. 37	24. 7

^{*} Illumination 1.85 X 10⁻² lumens - ft²

Resolution 50 ft. (500 lines).

Range 100, 000 ft.

Focal length = 9.15 cms

The results of table VIII are plotted in figure 18.

Table IX shows S/N vs. frame rate under sunlight illumination of 2.77×10^2 lumens - ft^2 . The sun is assumed to be at an elevation of 3° .

TABLE IX*

S/N vs. frame rate 3° sun elevation

cps 5 10 15 20	S/N
cps	RMS Current
5	104
10	7.13 X 10 ³
15	5. 82 X 10 ³
20	5. 03 X 10 ³
25	4. 5 X 10 ³
30	4.1 X 10 ³

^{*} clear weather α = 0.6, γ o = 0.9, γ = 0.2, γ = 0.1 Lens diameter = 9.15 cms γ = 0.5 μ

Range 100, 000 ft.

The results of Table IX are plotted in figure 20.

Table X shows the resolution obtainable vs. lens diameter under moonlight illumination for a S/N of 2 (RMS current)

TABLE X*

Resolution vs. lens diameter moonlight illumination

Resolution	Lens F	Lens diamete					
lines	number	cm s					
100	0. 83	11					
200	0. 59	15. 6					
300	0.48	19					
400	0. 42	21. 9					
500	0. 37	24. 7					

* Illumination: 1.85 X 10^{-2} lumens - ft^2 S/N = 2, γ o = 0.9, R_r = 0.2, R_b = 0.1 frame rate 30 cps, lens focal length 9.15 cms

The results of table X are plotted in fig. 21.

Table XI indicates the relationship between frame rate and lens diameter with the sun elevation at 3° and with an object resolution of 10 ft. The lens focal length has been restricted to 46 cms.

TABLE XI*
Lens diameter vs. frame rate

Frame rate	Lens F	Lens Diameter
cps	Number	cms
5	71.5	0. 64
10	59. 8	0. 77
15	54	0. 85
20	50	0. 915
25	47.5	0. 97
30	45. 5	1. 01

*Illumination 227 lumens - ft²

lens focal length 46 cms

$$\lambda = 0.5 \,\mu$$
, $R_r = 0.2$, $R_B = 0.1$

$$\eta_{0} = 0.9, \alpha = 0.6$$

The results of table XI are plotted in figure 22.

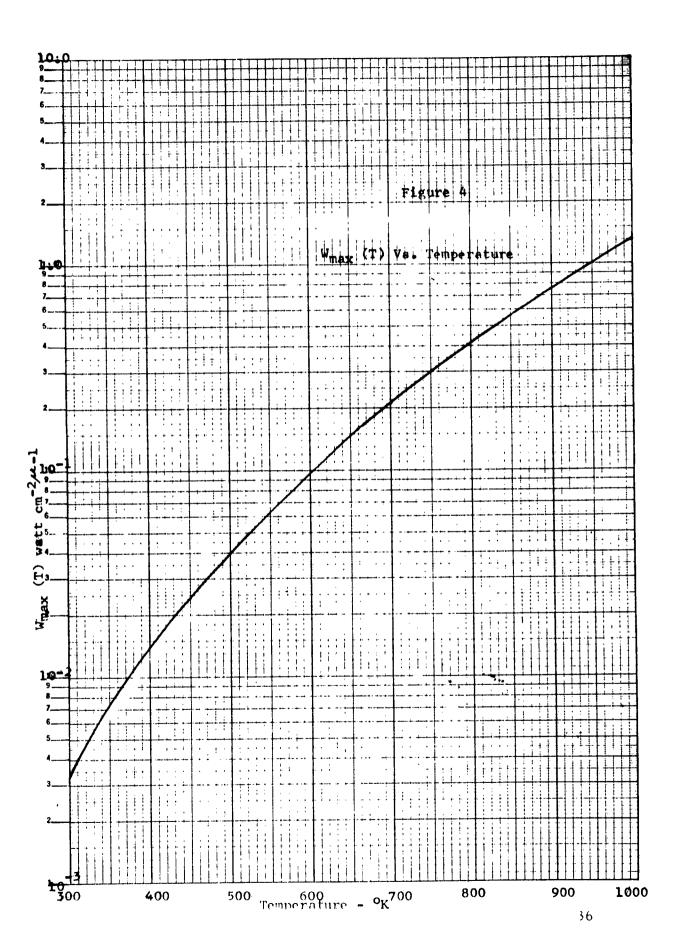
Table XII shows the relation between lens diameter vs. S/N for a 3° sun elevation and for an object resolution of 10 ft.

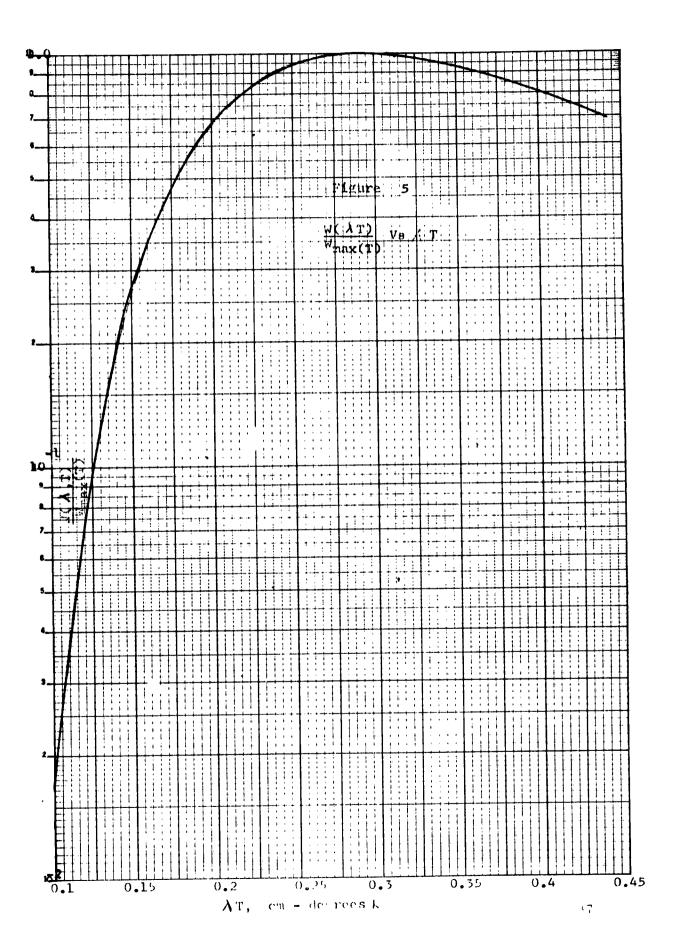
TABLE XII*
S/N vs. Lens Diameter

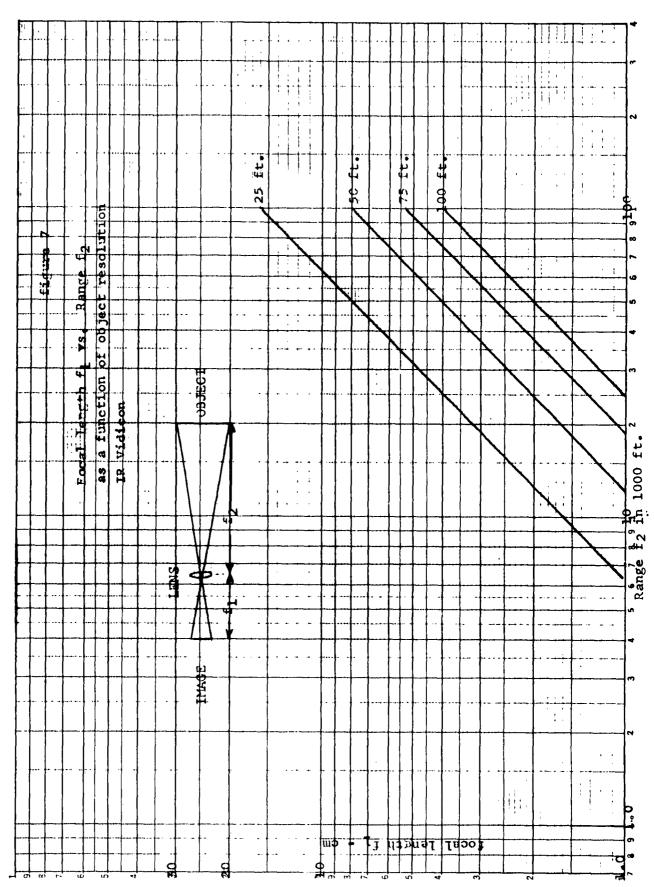
S/N	Lens F	Lens Diameter
RMS Current	Number	cm s
2	45	1. 01
4	32	1. 43
8	23	2. 03
16	16	2. 87
32	11.3	4. 07
64	8	5. 75
100	6.4	7.18
128	5.7	8. 1
200	4.6	10

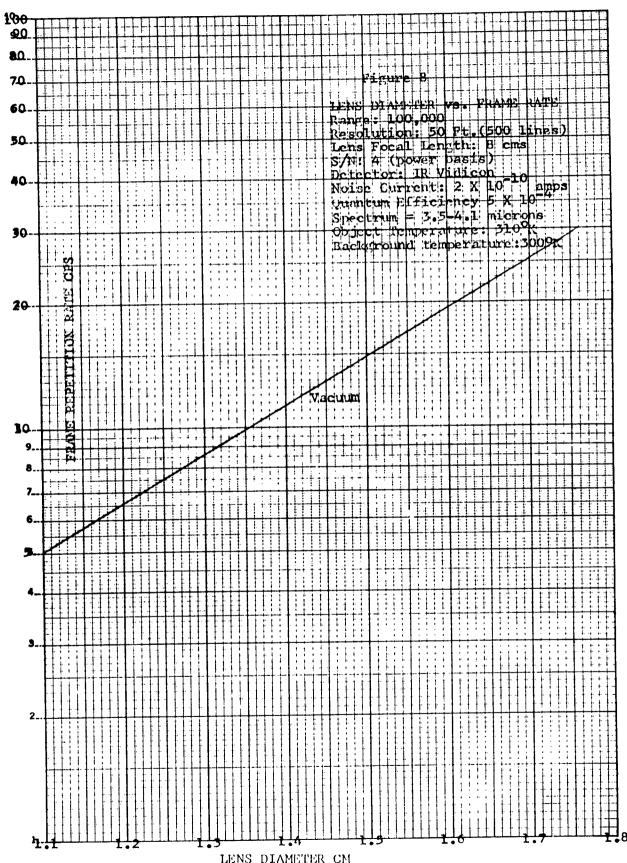
^{*}Focal length 46 cms, f_r = 30 cps, R_r = 0.2, R_B = 0.1, η 0 = 0.9, α 0 = 0.6, α 0 = 0.5 α 0.

The results of table XII are plotted in figure 23.

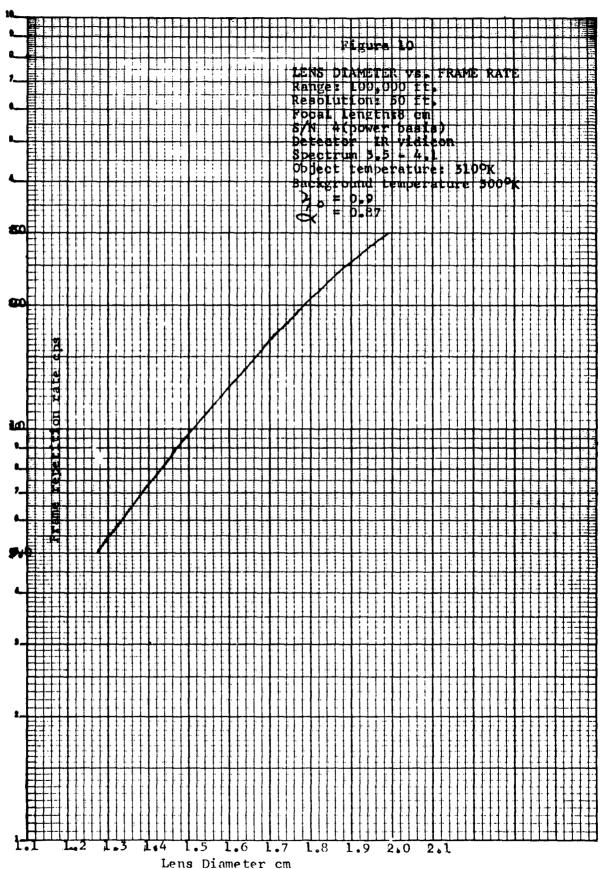


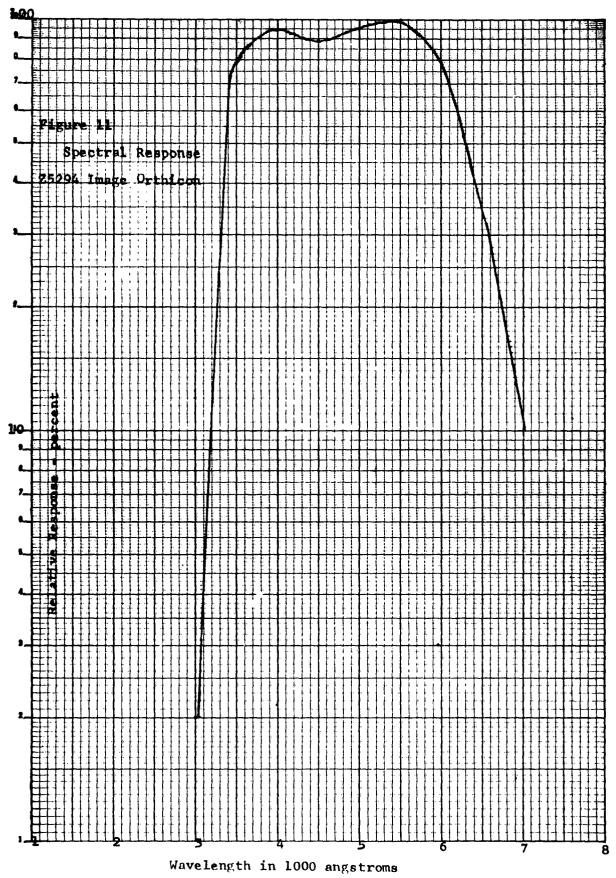


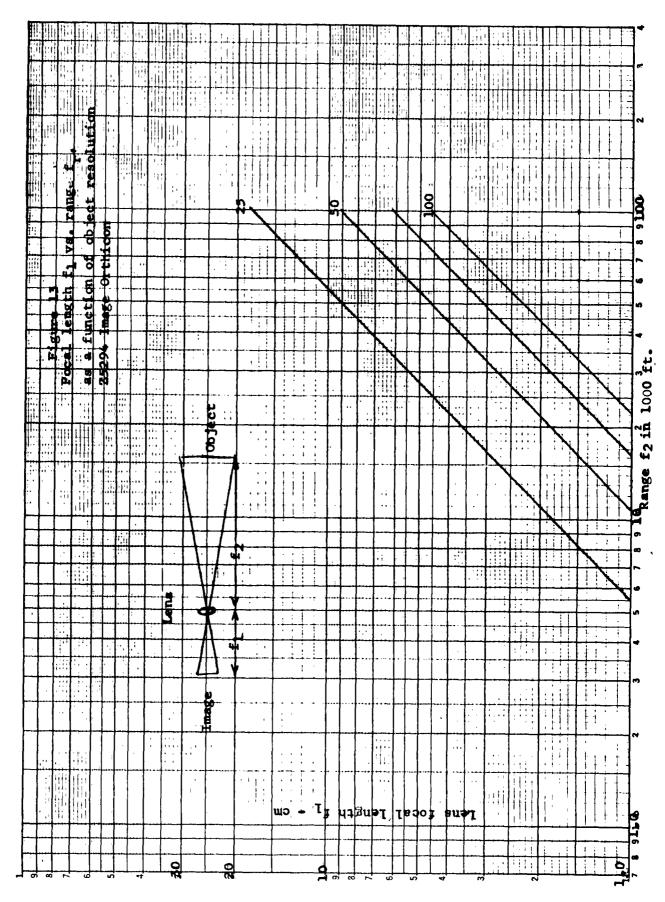


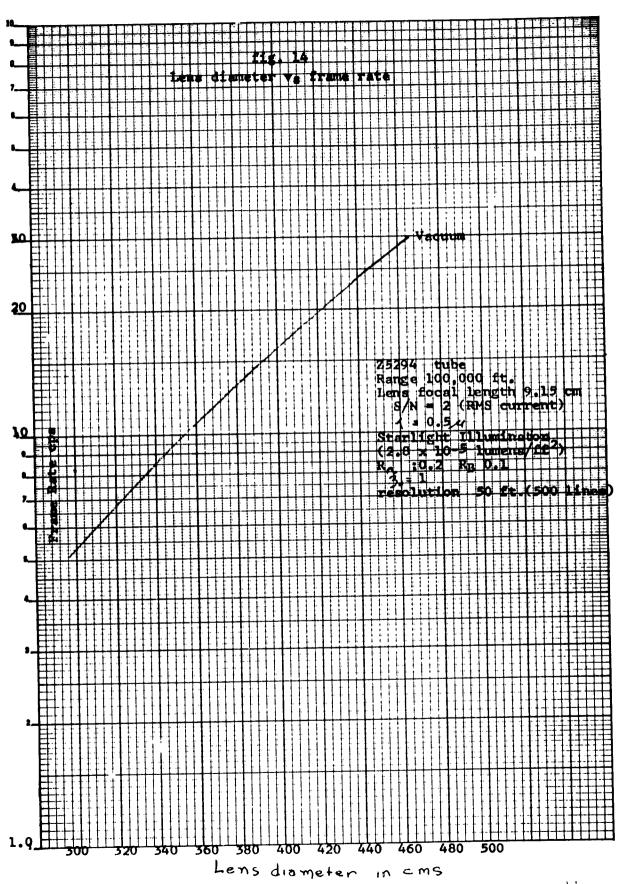


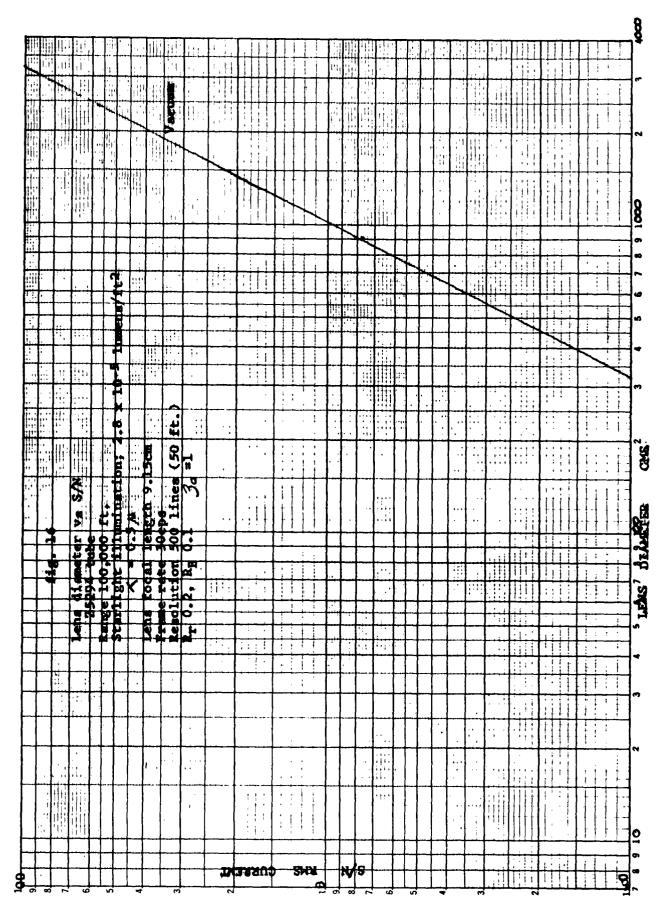
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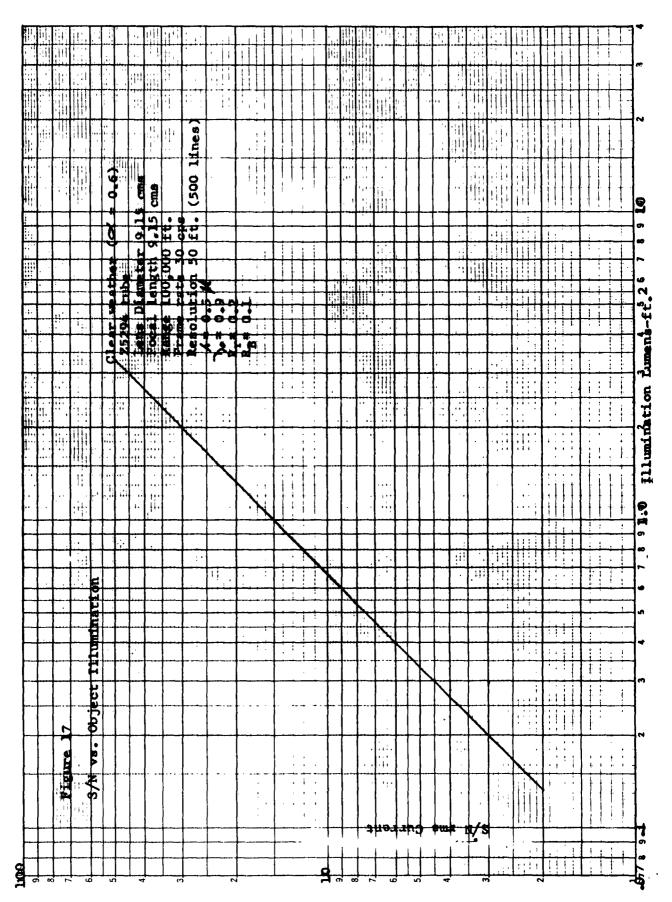


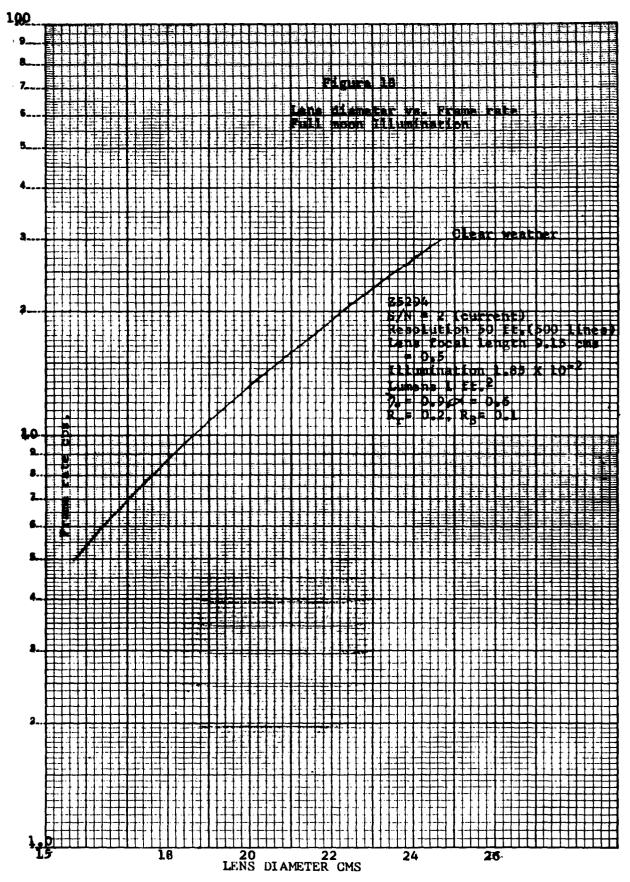


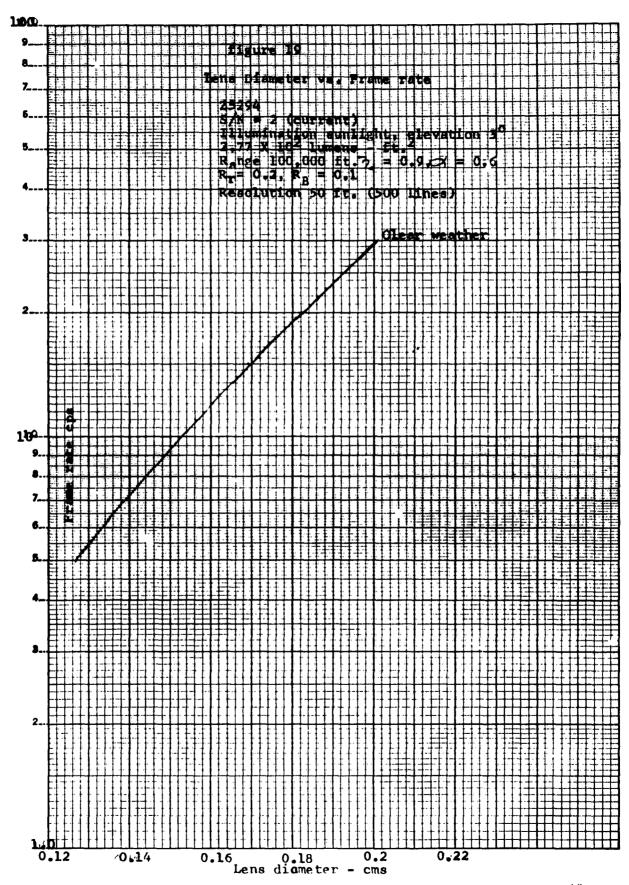


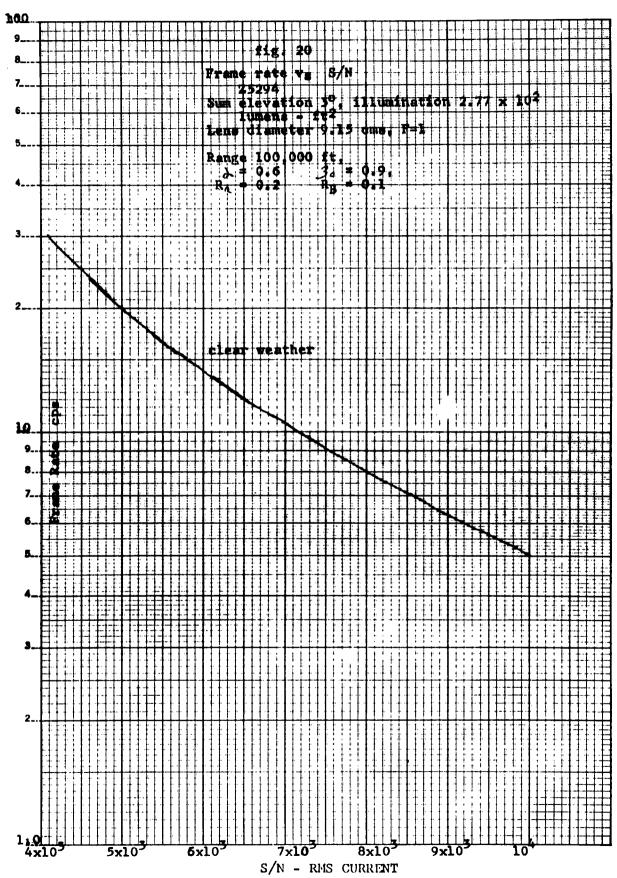


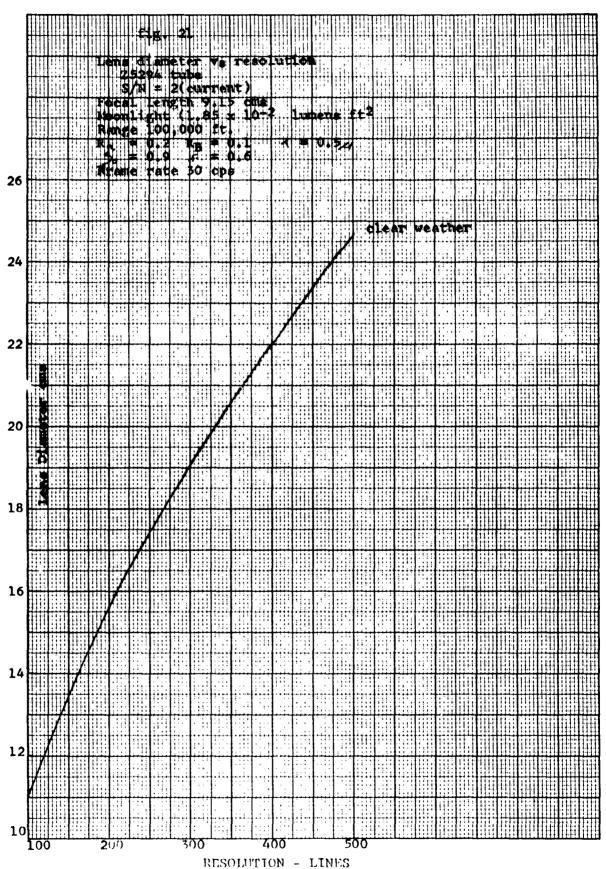


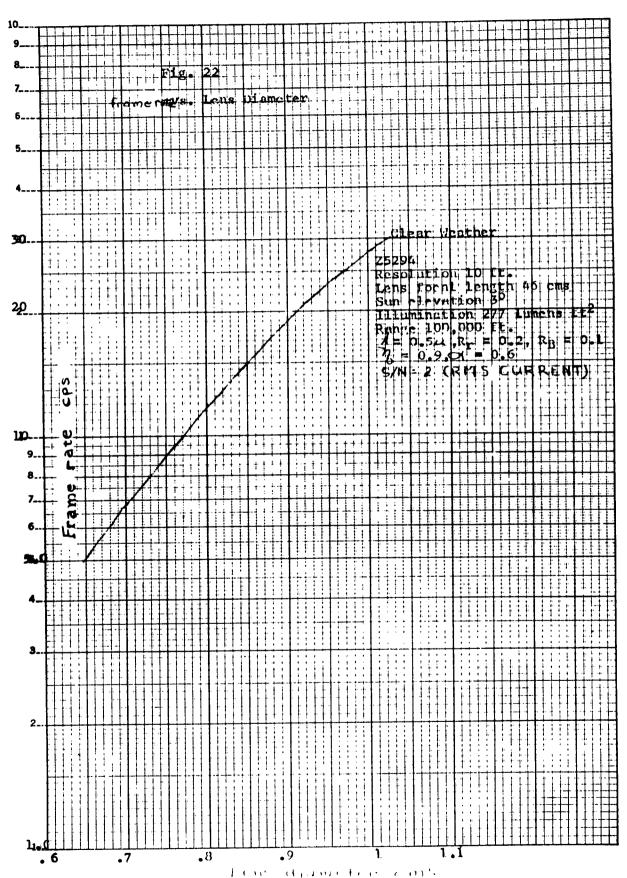


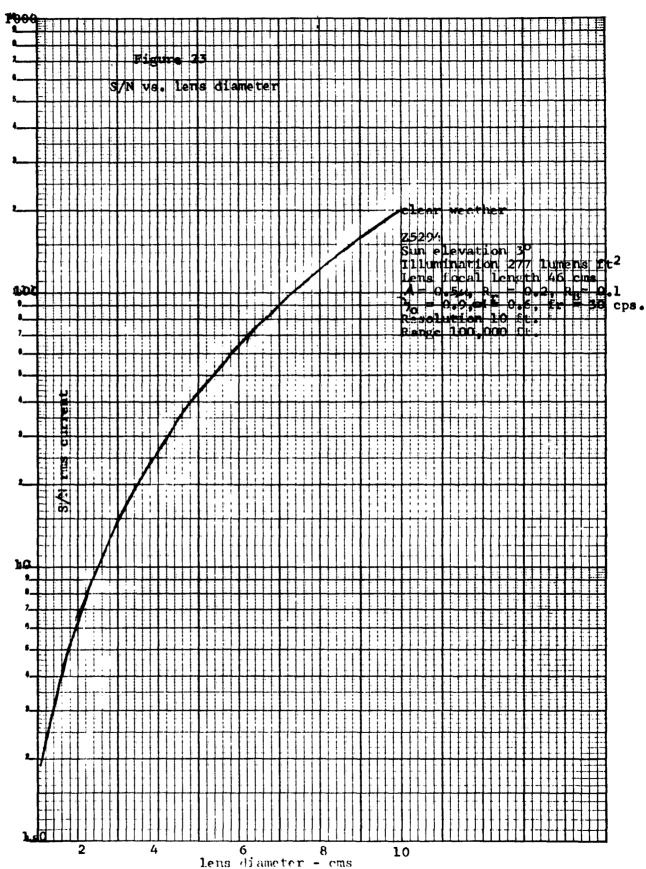


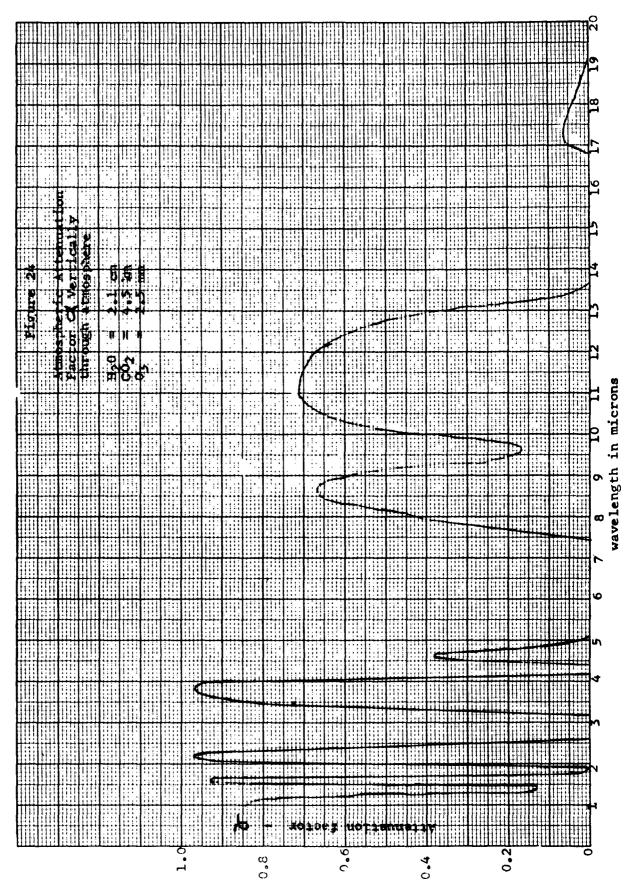












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